Subject: Unbalanced Magnetic Flux for Half Turn Winding Design

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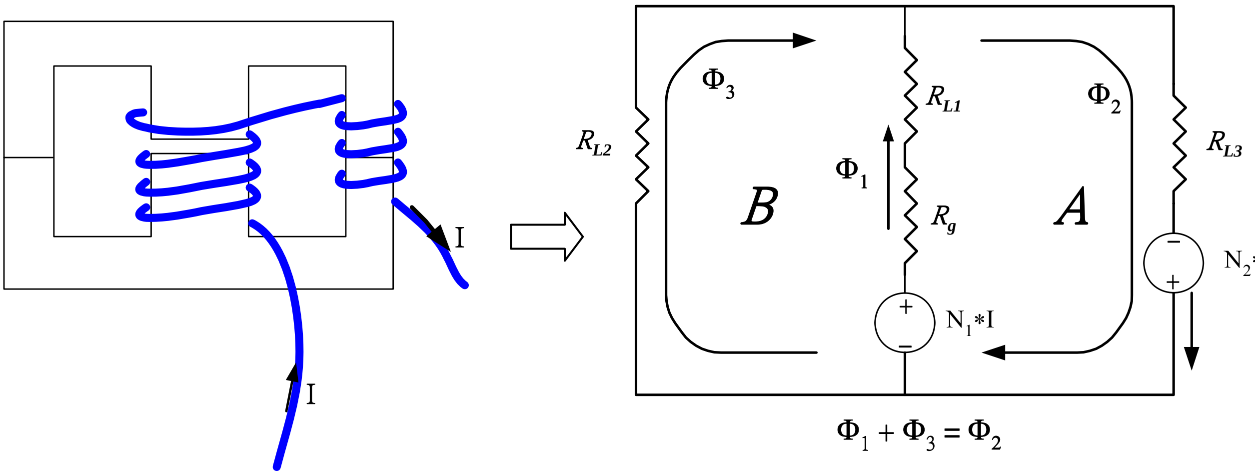


The above winding structure leads to unbalanced magnetic flux distribution in the symmetric core set. By analogy to electric circuit, the magnetic circuit can be formed to get the flux solutions for each magnetic path by using Kirchhoff′s laws. We can get the flux equation first:

$$\Phi_1 + \Phi_3 = \Phi_2 \qquad (1)$$

If considering the mesh *A*, we can get the second equation:

$$(\boldsymbol{N}_1 + \boldsymbol{N}_2)\cdot \boldsymbol{I} = \Phi_1 \cdot (\boldsymbol{R}_{L1} + \boldsymbol{R}_g) + \Phi_2 \cdot \boldsymbol{R}_{L3} \qquad (2)$$

in which $\boldsymbol{N}_1$ is the winding turns of the center pole and $\boldsymbol{N}_2$ the winding turns of the outer leg and the formed pole direction of the magnetomotive force, $\boldsymbol{N}\cdot\boldsymbol{I}$, is defined by the winding and current direction. $\boldsymbol{R}_g$ is the reluctance of the center gap. $\boldsymbol{R}_{L1}$, $\boldsymbol{R}_{L2}$ and $\boldsymbol{R}_{L3}$ are simplified representation of the segment reluctance on the core.

If considering the mesh *B*, we can get the third equation:

$$\boldsymbol{N}_1 \cdot \boldsymbol{I} = \Phi_1 \cdot (\boldsymbol{R}_{L1} + \boldsymbol{R}_g) - \Phi_3 \cdot \boldsymbol{R}_{L2} \qquad (3)$$

With these three equations the flux Φ_1, Φ_2 and Φ_3 can be obtained. Due to the fact that the fluxes linking winding $\boldsymbol{N}_1$ and winding $\boldsymbol{N}_2$ are different in most of the cases, the inductance should be expressed as $\boldsymbol{L} = \dfrac{\Sigma \boldsymbol{N}_i \cdot \Delta\Phi_i}{\Delta \boldsymbol{I}}$ (4)

If $\boldsymbol{N}_2$= 0, the concerned question will be simplified to solve the symmetric flux flow through the symmetric core set. The core factors, $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$, are normally based on this kind of balanced winding and magnetic path to be calculated. The core factor $\boldsymbol{C}_1$ is defined by:

$$C_1 = \sum_i \frac{l_i}{A_i} \qquad (5)$$

and the reluctance of a segment is: $\boldsymbol{R}_i = \dfrac{\boldsymbol{l}_i}{\mu_0 \cdot \mu \cdot \boldsymbol{A}_i} = \dfrac{\boldsymbol{r}_i}{\mu_0 \cdot \mu}$ where $\boldsymbol{r}_i = \boldsymbol{l}_i/\boldsymbol{A}_i$ is called reluctance factor of magnetic segment i and μ_o=4πx10^{-7}. Back to the previous unbalanced winding problem and assuming the gapping condition $\boldsymbol{R}_g$= 0 for simplicity, the calculated fluxes are obtained:

$$\Phi_1 = \frac{(2\boldsymbol{N}_1 + \boldsymbol{N}_2)\cdot \boldsymbol{I}}{\boldsymbol{R}_L + 2\boldsymbol{R}_{L1}}$$

$$\Phi_2 = \left[\frac{\boldsymbol{N}_1 \cdot \boldsymbol{R}_L + \boldsymbol{N}_2 \cdot (\boldsymbol{R}_L + \boldsymbol{R}_{L1})}{\boldsymbol{R}_L \cdot (\boldsymbol{R}_L + 2\boldsymbol{R}_{L1})}\right]\cdot \boldsymbol{I}$$

$$\Phi_3 = \left[\frac{(2\boldsymbol{N}_1 + \boldsymbol{N}_2)\cdot \boldsymbol{R}_{L1} - \boldsymbol{N}_1 \cdot (\boldsymbol{R}_L + 2\boldsymbol{R}_{L1})}{\boldsymbol{R}_L \cdot (\boldsymbol{R}_L + 2\boldsymbol{R}_{L1})}\right]\cdot \boldsymbol{I}$$

where $\boldsymbol{R}_{L2} = \boldsymbol{R}_{L3} = \boldsymbol{R}_L$ by considering the symmetric core geometry. Considering the symmetric condition and $\boldsymbol{N}_2$= 0, the core factor $\boldsymbol{C}_1$ can be calculated by (5) as follows,

$$\boldsymbol{C}_1 = \mu_0 \cdot \mu \cdot \frac{1}{2}(\boldsymbol{R}_L + 2\boldsymbol{R}_{L1}) \quad \text{or} \quad \boldsymbol{R}_L + 2\boldsymbol{R}_{L1} = \frac{2\boldsymbol{C}_1}{\mu_0 \cdot \mu}$$

Substituting the result in the flux Φ_1, Φ_2 and Φ_3 and denoting $\boldsymbol{r}_{L1} = \dfrac{\boldsymbol{l}_{L1}}{\boldsymbol{A}_{L1}}$,

we then can find the flux equations as follows,

$$\Phi_1 = \mu_0 \cdot \mu \frac{(2\boldsymbol{N}_1 + \boldsymbol{N}_2)\cdot \boldsymbol{I}}{2\boldsymbol{C}_1}$$

$$\Phi_2 = \mu_0 \cdot \mu \left[\frac{\boldsymbol{N}_1 \cdot (2\boldsymbol{C}_1 - 2\boldsymbol{r}_{L1}) + \boldsymbol{N}_2 \cdot (2\boldsymbol{C}_1 - \boldsymbol{r}_{L1})}{2\boldsymbol{C}_1 \cdot (2\boldsymbol{C}_1 - 2\boldsymbol{r}_{L1})}\right]\cdot \boldsymbol{I}$$

$$\Phi_3 = \mu_0 \cdot \mu \left[\frac{(2\boldsymbol{N}_1 + \boldsymbol{N}_2)\cdot \boldsymbol{r}_{L1} - \boldsymbol{N}_1 \cdot 2\boldsymbol{C}_1}{2\boldsymbol{C}_1 \cdot (2\boldsymbol{C}_1 - 2\boldsymbol{r}_{L1})}\right]\cdot \boldsymbol{I}$$

Remark:
$\boldsymbol{l}_{L1}$ and $\boldsymbol{A}_{L1}$ are the length and cross sectional area of the center post. μ is the initial permeability of the core material.

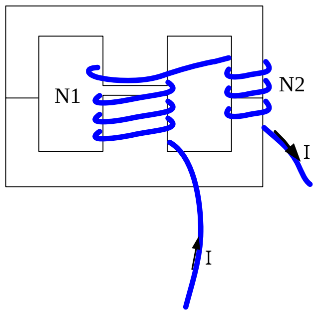
Accordingly the inductance can be calculated to get the following equation:

$$L=\frac{\Sigma N_i\cdot\Delta\Phi_i}{\Delta I}=\mu_0\cdot\mu\left[\frac{N_1(2N_1+N_2)}{2C_1}+\frac{N_1\cdot N_2(2C_1-2r_{L1})+N_2^2(2C_1-r_{L1})}{2C_1\cdot(2C_1-2r_{L1})}\right] \qquad (6)$$

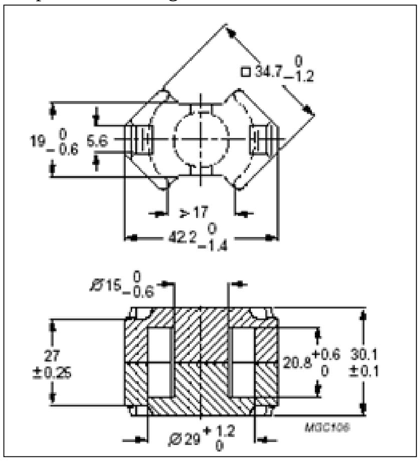
The measurement is then made on the core type of RM14/I-3C90 to check if the calculated result according to the above conclusion (6) is consistent with the reality. Under the consideration of parasitic gap between the matting faces of the core set which is about 6.2μm, the permeability μ=1,900 is then input for the inductance calculation. The comparison data are shown as below :

N1: winding of center post, N2: winding of outer leg

N1	N2	$L(\mu H)_{calcul}$	$L(\mu H)_{measured}$
0	1	4.30	4.5
1	0	6.76	6.9
1	1	17.82	18.3
2	0	27.05	27.0
2	1	44.87	44.8
3	0	60.86	60.4
3	1	85.44	84.8
3	2	118.62	119.0



RM14/I product drawing:



For RM14/I, C_1=0.353mm^{-1}, r_{L1}= 0.124mm^{-1}

$\mu_0\cdot\mu = 2.387*10^{-3}$

$$L=\frac{\Sigma N_i\cdot\Delta\Phi_i}{\Delta I}=\mu_0\cdot\mu\left[\frac{N_1(2N_1+N_2)}{2C_1}+\frac{N_1\cdot N_2(2C_1-2r_{L1})+N_2^2(2C_1-r_{L1})}{2C_1\cdot(2C_1-2r_{L1})}\right]$$

The good fit of the calculation result to the measured data leads to the conclusion that the methodology of magnetic circuit analogy to electric circuit adopted to solve the general inductance equation (6) is workable and effective. It should be noted that winding N_1 and N_2 in (6) could be input as negative integer to represent the opposite pole direction of the concerned magnetomotive force in the magnetic circuit.

When the reluctance R_g of the center gap of the core set can not be ignored anymore, the flux calculation will become more complicated. Firstly the air-gap reluctance has to be written as follows,

$$R_g=\frac{l_g}{\mu_0\cdot\mu\cdot A_g}=\frac{r_g}{\mu_0} \qquad (7)$$

in which the relative permeability μ=1 for air and $r_g=l_g/A_g$ is denoted as the air-gap reluctance factor with l_g the gap length and A_g the cross area for the core face forming the gap. The resulted flux equations can be obtained as follows,

$$\Phi_1 = \mu_0 \cdot \mu \frac{(2\boldsymbol{N}_1 + \boldsymbol{N}_2)\cdot \boldsymbol{I}}{2(\boldsymbol{C}_1 + \mu \cdot \boldsymbol{r}_g)}$$

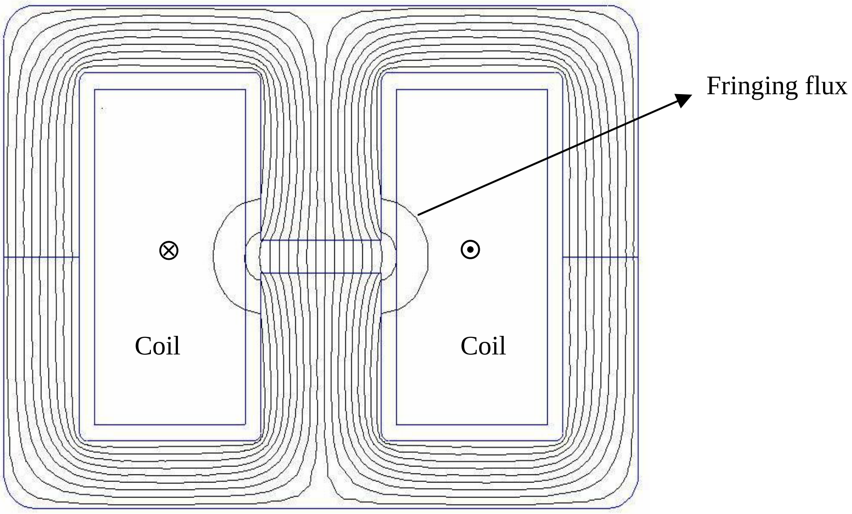
$$\Phi_2 = \mu_0 \cdot \mu \left[\frac{2\boldsymbol{N}_1 \cdot (\boldsymbol{C}_1 - \boldsymbol{r}_{L1}) + \boldsymbol{N}_2 \cdot (2\boldsymbol{C}_1 - \boldsymbol{r}_{L1} + \mu \cdot \boldsymbol{r}_g)}{4(\boldsymbol{C}_1 + \mu \cdot \boldsymbol{r}_g)\cdot(\boldsymbol{C}_1 - \boldsymbol{r}_{L1})} \right] \cdot \boldsymbol{I}$$

$$\Phi_3 = \mu_0 \cdot \mu \left[\frac{2\boldsymbol{N}_1 (\boldsymbol{r}_{L1} - \boldsymbol{C}_1) + \boldsymbol{N}_2 \cdot (\boldsymbol{r}_{L1} + \mu \cdot \boldsymbol{r}_g)}{4(\boldsymbol{C}_1 + \mu \cdot \boldsymbol{r}_g)\cdot(\boldsymbol{C}_1 - \boldsymbol{r}_{L1})} \right] \cdot \boldsymbol{I}$$

Again, μ shown in the above flux equations is the initial permeability of the core material. Similarly by taking account of the original inductance definition by reference to (4), the calculation can be worked out to get the following inductance equation:

$$\boldsymbol{L} = \frac{\Sigma \boldsymbol{N}_i \cdot \Delta\Phi_i}{\Delta \boldsymbol{I}} = \mu_0 \cdot \mu \left[\frac{\boldsymbol{N}_1(2\boldsymbol{N}_1 + \boldsymbol{N}_2)}{2(\boldsymbol{C}_1 + \mu \cdot \boldsymbol{r}_g)} + \frac{2\boldsymbol{N}_1 \cdot \boldsymbol{N}_2(\boldsymbol{C}_1 - \boldsymbol{r}_{L1}) + \boldsymbol{N}_2^2(2\boldsymbol{C}_1 - \boldsymbol{r}_{L1} + \mu \cdot \boldsymbol{r}_g)}{4(\boldsymbol{C}_1 + \mu \cdot \boldsymbol{r}_g)\cdot(\boldsymbol{C}_1 - \boldsymbol{r}_{L1})} \right] \quad (8)$$

It should be noted that the effect of air-gap on magnetic flux can be observed by the following computed plot of flux distribution by ***FEMM*** in and around the gapped pair of E-like cores:



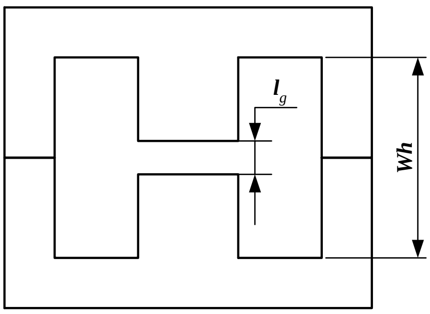
The fringing flux reduces the reluctance of the gap due to the fact that the effective area of the gap is larger than the cross area $\boldsymbol{A}_g$ of the core face forming the gap. In the case of the above gapped pair of E cores, $\boldsymbol{A}_g$ is just of the center post. It is then required to introduce a compensation factor, called fringing factor, $\boldsymbol{F}(\geqq 1)$ so as to properly depict the realized reluctance factor $\boldsymbol{r}_g$ of the air gap by rewritting (7) as the following equation:

$$r_g = \frac{l_g}{A_g \cdot F} \qquad (9)$$

Since the fringing flux is not simply a linear behavior in relation to the gap size, it is, however, not an easy job to work on the F factor. "*Any attempt to analyse the problem on a general basis is complicated by the fact that, as the gap becomes larger, the fringing or leakage flux becomes dependent not only on the gap geometry but also on that of the winding and the rest of the magnetic circuit.*" quoted from E.C. Snelling. However, a lot of efforts were still made by researchers trying to formulate the fringing factor F. One of the empirical results issued by the laboratory of former Philips Magnetic Components (now branded as Ferroxcube) can be worthy to adopt:

$$F = 1 + \frac{l_g}{\sqrt{A_g}} \cdot \ln\left(\frac{2 \cdot Wh}{l_g}\right) \qquad (10)$$

in which Wh is the window height of the core set as shown in the below drawing:



With (9) and (10) the numerical calculation on the inductance equation (8) and the concerned flux equations can be executed. One can build up a simple numerical claculation model as below,

Input data in the green areas and then you will find the calculated values in pink areas.

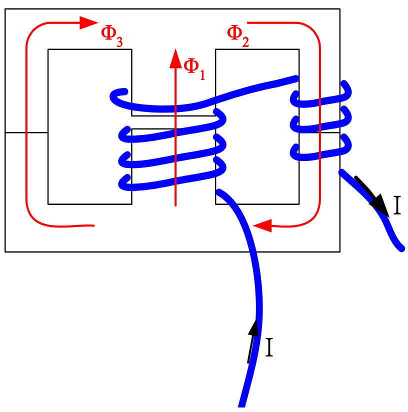
N1: winding of center post, N2: winding of outer leg

N1	N2	$L(\mu H)_{calcul}$	Φ_1(Weber)	Φ_2(Weber)	Φ_3(Weber)	B_1(mT)	B_2(mT)	B_3(mT)
3	2	20.17	7.527E-06	2.904E-05	2.152E-05	44.35	241.42	178.85

Remark:
The segment flux density is defined as $B_i = \Phi_i / A_i$

I_{DC}=	4.00	Ampere	
μi=	2300		→ Initial permeability
C_1=	0.35	mm^{-1}	
l_{L1}=	21.10	mm	→ Wh
A_{L1}=	169.7	mm^2	→ A_g

effective area Ae=	198	mm^2
effective length le=	70	mm
cross sectional area of center post=	169.7	mm^2
cross sectional area of outer leg=	120.3	mm^2
gap length l_g=	0.5	mm



Again, the core type RM14/I-3C90 is selected to get the core geometry parameters as the input data by taking account of the gap length l_g=0.5mm. The effect of fringing flux due to the gap is shown in the below calculation:

μe=	132.34		→ Effective permeability
μo*μi=	2890	$x10^{-6}$	
r_{L1}=	0.12	mm^{-1}	→ Reluctance factor of the center leg segment of the core set
F =	1.17		→ Fringing flux factor
r_g =	0.0025	mm^{-1}	→ Reluctance factor of the gap segment

If one inputs the gap length l_g=6.2μm by assuming ungapped condition and taking account of the parasitic gap between the matting faces, almost the same result as the one shown in the first example calculated by (6) can be observed.

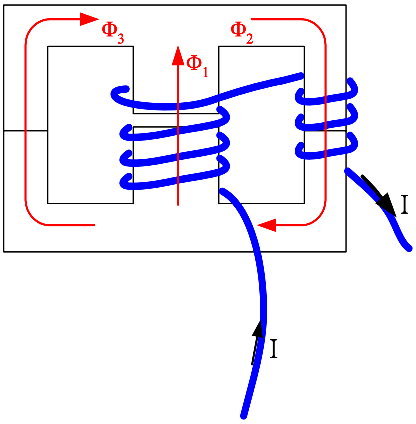
One can do the inverse winding direction on N_2 by inputting negative integer, -2, for interest to see how the flux flows in each magnetic segment are changed:

Input data in the green areas and then you will find the calculated values in pink areas.

N1: winding of center post, N2: winding of outer leg

N1	N2	$L(\mu H)_{calcul}$	Φ_1(Weber)	Φ_2(Weber)	Φ_3(Weber)	B_1(mT)	B_2(mT)	B_3(mT)
3	-2	14.52	3.763E-06	-2.340E-05	-2.716E-05	22.18	-194.50	-225.78

As a result of that, the negative numerical data of Φ_2 and Φ_3 can be obtained and the minus sign indicates the flux flow direction is opposite to the arrow direction marked in red line of the flux shown in the drawing:



The above examples of numerical calculation with "half turn" winding due to $N_2 \neq 0$ lead to unbalanced magnetic flux: $\Phi_2 \neq \Phi_3$. Moreover, the winding direction of N_2 also makes the highest loaded flux be able to be selected to the specified magnetic segment. With the help of the above simple calculation model "half turn" winding design becomes possible and feasible with ease as the concerned magnetic flux behavior can be predicted in a simple way.

Though some powerful computer-aided softwares for 2-D or 3-D electromagnetic field simulation in use of finite element or boundary element method are available, only experienced or well trained operators can do the right job. This is so because the complicated input parameters for the soft programs are not easy to be judged by junior users. Any wrong input parameter will lead to exaggerated and incorrect result.

Conventional magnetic component design has been confronted with the bottleneck of existing soft magnetic materials and ordinary winding methods as well as lack of innovation. As a result of that, bloody price competition is the fate. Departing from the traditional winding design might give rise to one of the ways toward **product innovation**.

About the author:

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