Bull Will

Model Analysis of Transformers

Advanced training course C

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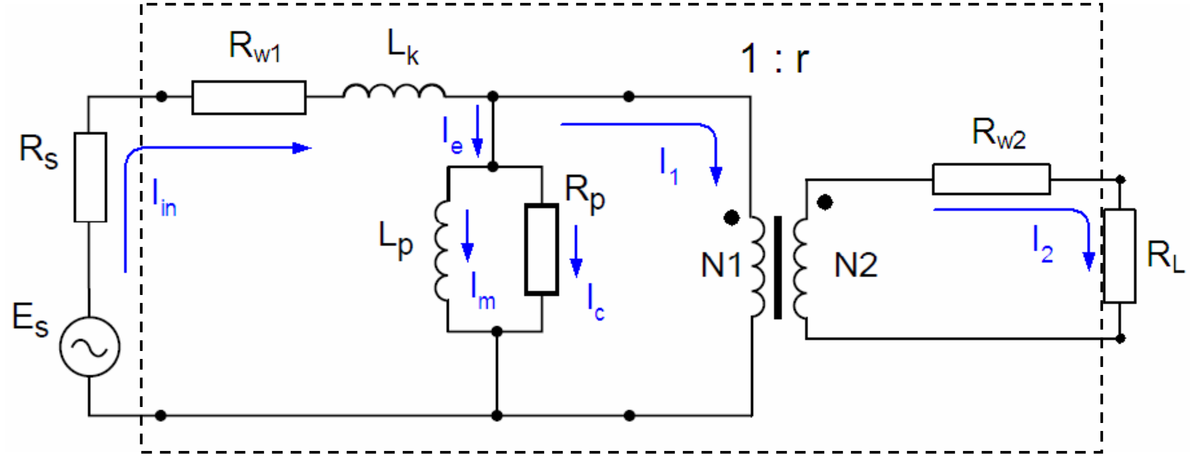
- **Equivalent circuit of a transformer**



E_s: Voltage source

R_s: Source impedance

R_{w1}: Primary winding resistance

L_k: Total leakage inductance

L_p: Primary self inductance in parallel mode

R_p: Core loss resistance in parallel mode

R_{w2}: Secondary winding resistance

R_L: Load resistance

N1: Primary winding turns

N2: Secondary winding turns

r: Turn ratio

I_{in}: Input current

I_1: Reflected current from I_2

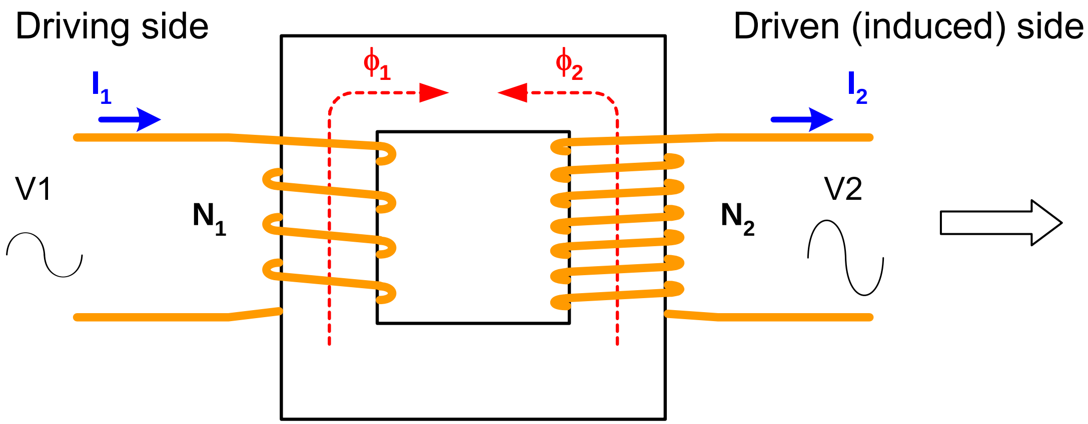
I_2: Load current

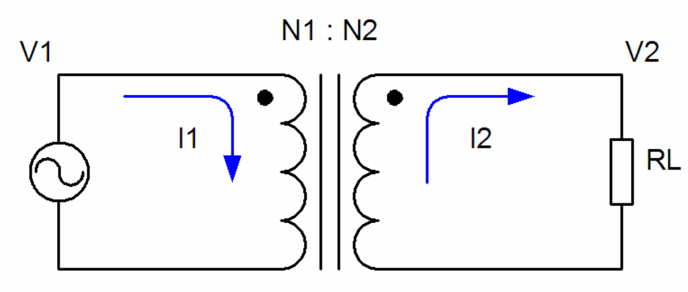
I_e: Exciting current

I_m: Magnetizing current

I_c: Core loss current

• Ideal transformer theory





Polarity dot rule of a transformer circuit:

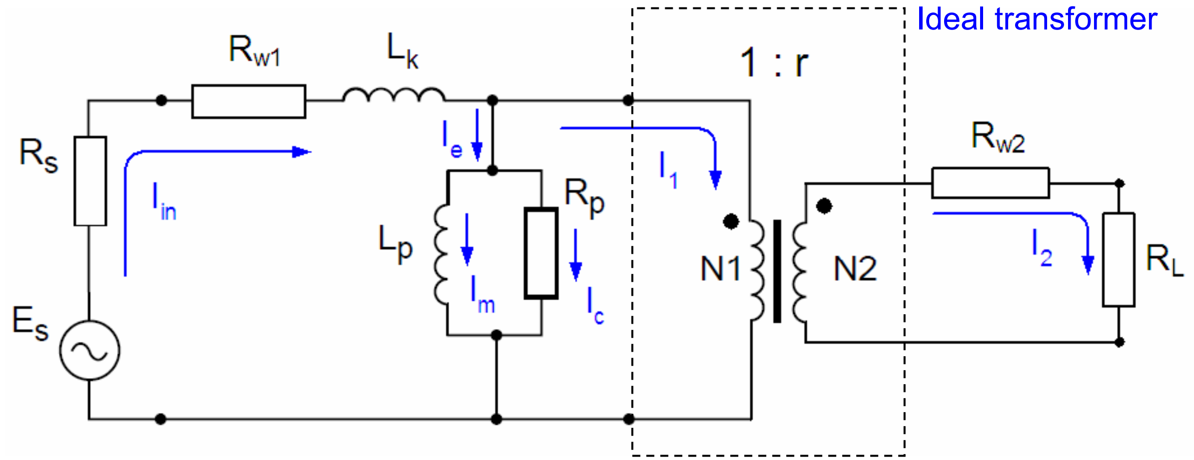
When I_1 flows into the primary dot side, then I_2 will flow out from the secondary dot side. Moreover, I_1 follows the waveform of I_2 and with the same phase.

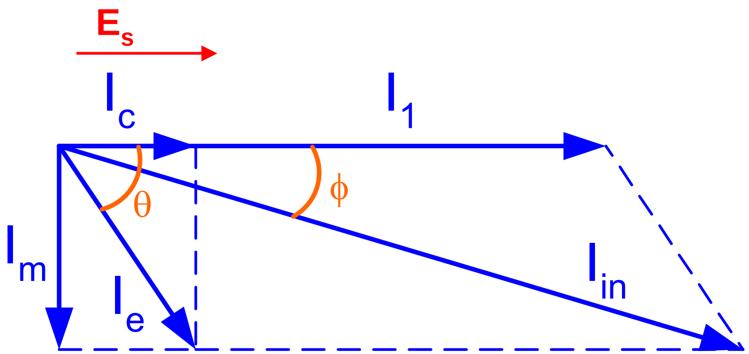
① $$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$ ⇒ 100% coupling, no flux leakage and stray capacitance

② $$N_1 \times I_1 = N_2 \times I_2$$ ⇒ Flux ϕ_1 driven by magneto-motive force N_1 x I_1 equals flux ϕ_2 driven by induced magneto-motive force N_2 x I_2 .

③ No winding loss and no core loss; L_p & $R_p \rightarrow \infty$

• Vector diagram of transformer currents

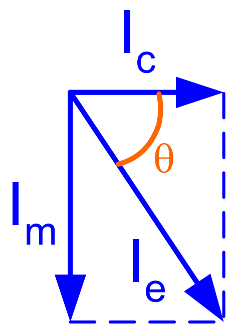


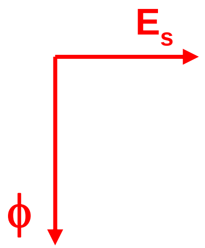


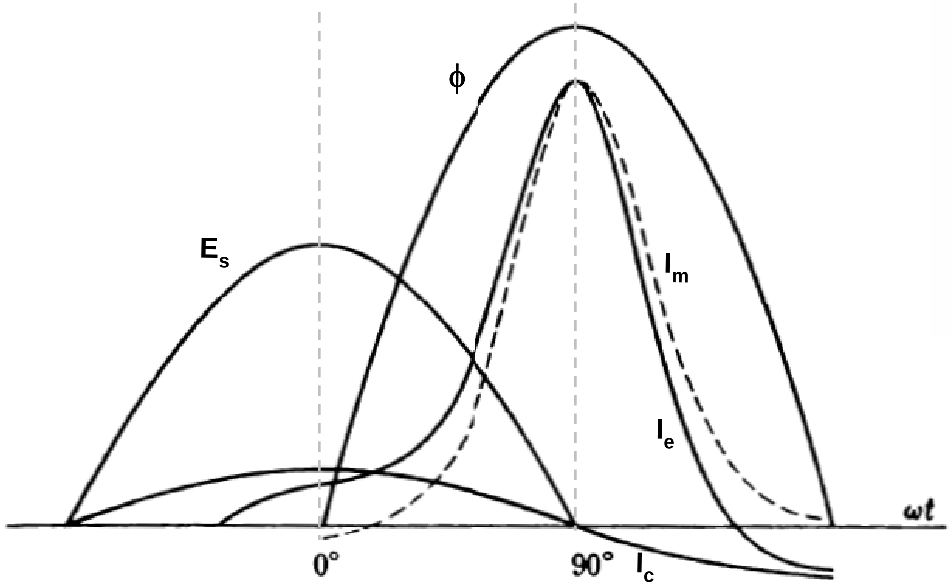
Vector set, $I_e = I_m + I_c$, is decided by the core material and the primary inductance. Ideally it's nothing to do with the reflected current $I_1 = r \times I_2$.

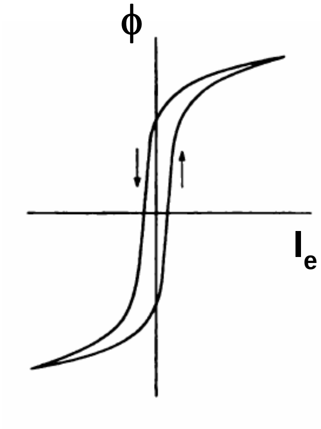
This vector set can be worked out by measuring the input current I_{in} and PF ($\cos\theta$) at open load condition.

• Waveform analysis of exciting current I_e – practical case





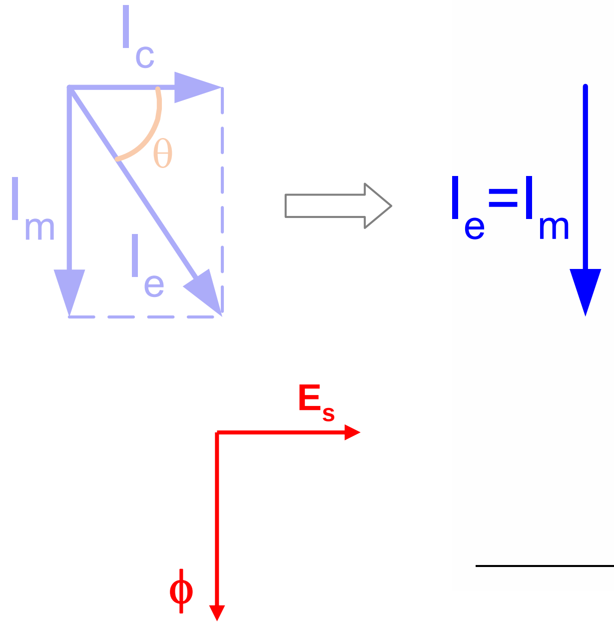


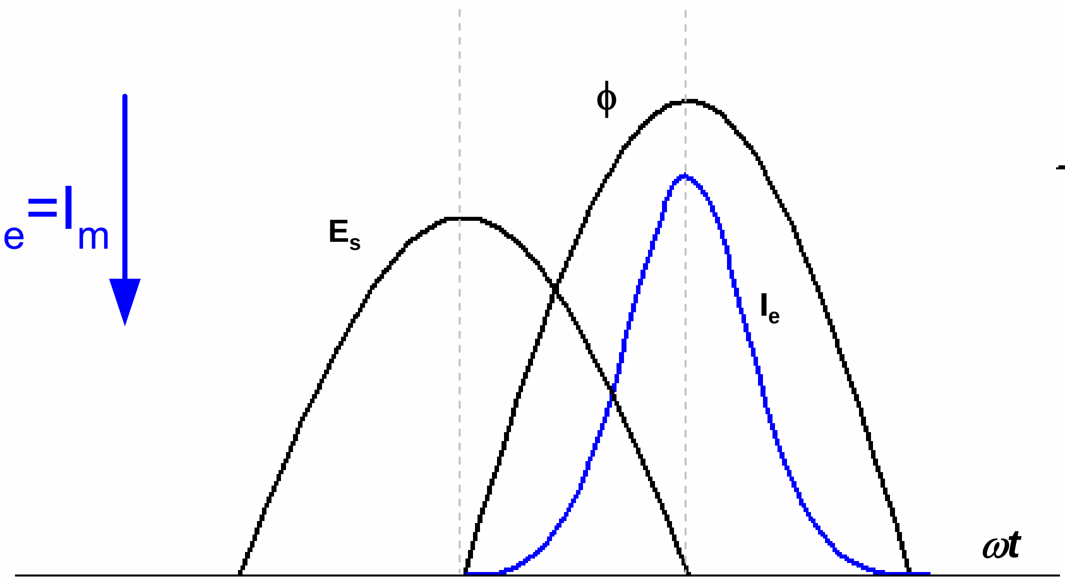


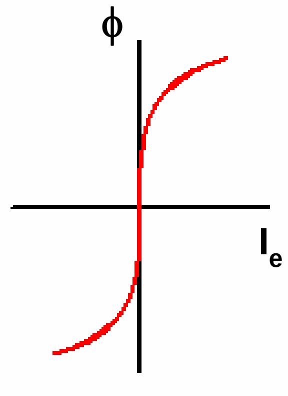
Hysteresis loop

If the applied voltage E_s is sinusoidal, the waveform of induced flux ϕ also is very nearly sinusoidal. However, the resulted exciting current will go non-sinusoidally with lots of harmonic components due to the hysteresis loop of the core material. The core loss current I_c is in phase with E_s and goes sinusoidally. Thus, the core loss Pc can be calculated by $\mathbf{E_s \cdot I_c}$.

- **Waveform analysis of exciting current – if ignoring core loss effect**

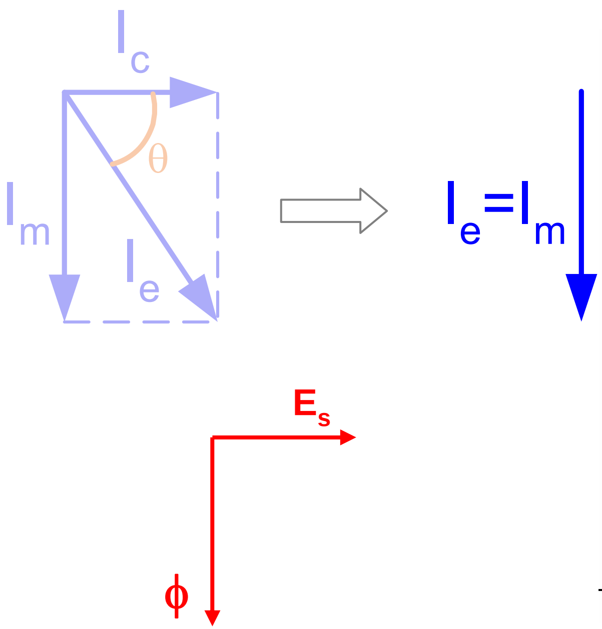


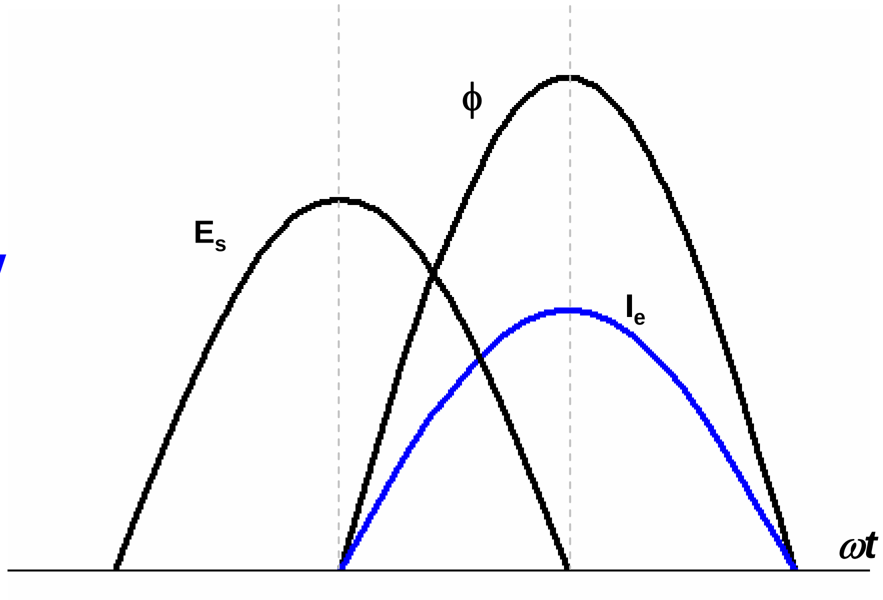


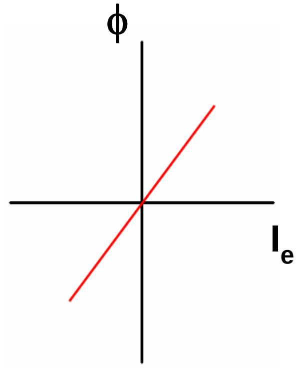


If the applied voltage E_s is sinusoidal, the waveform of induced flux ϕ also is very nearly sinusoidal. The resulted exciting current still will go non-sinusoidally with some harmonic components even if the hysteresis loop of the core material has no core loss.

- **Waveform analysis of exciting current – if B-H curve is linear**





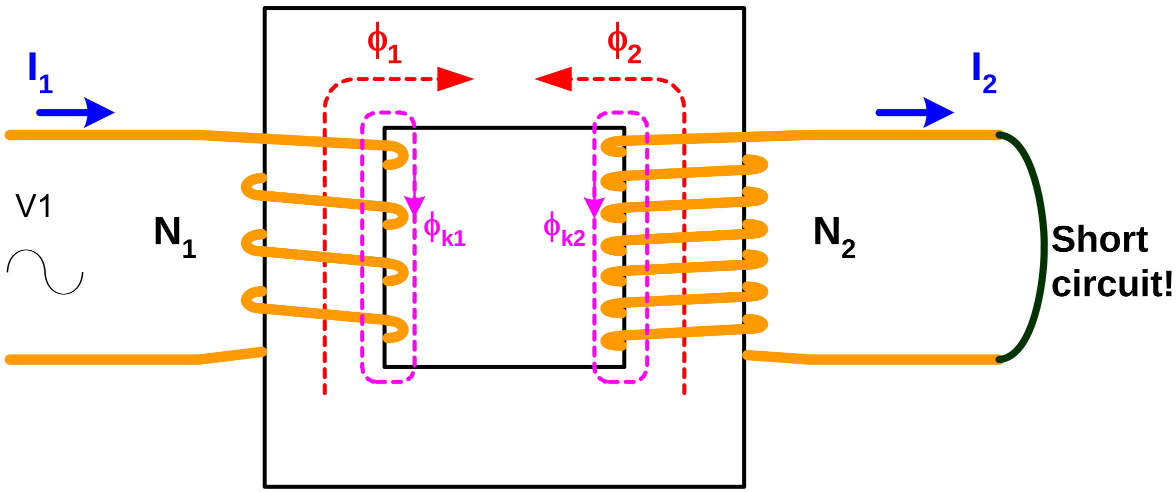


Hysteresis loop

If the applied voltage E_s is sinusoidal, the waveform of induced flux ϕ also is very nearly sinusoidal. The resulted exciting current will go sinusoidally if only the B-H curve of the core material is linear. This is typically the design case of isolation transformers for xDSL modem to reduce the THD (Total Harmonic Distortion) which is caused by the harmonic components of the exciting current.

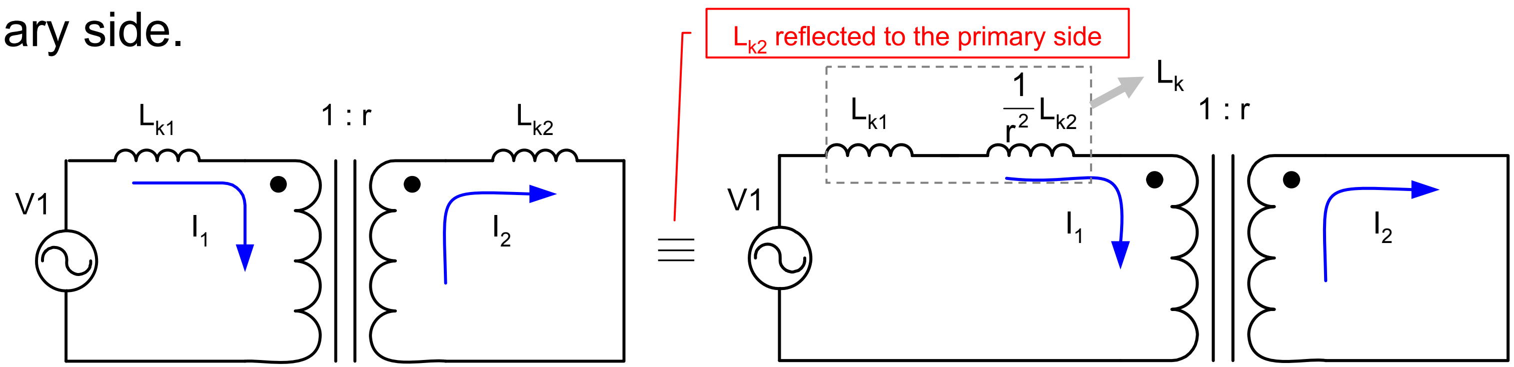
- **Leakage inductance L_k**

Ideally the flux ϕ_1 driven by magneto-motive force $N_1 \times I_1$ will be cancelled by flux ϕ_2 driven by induced magneto-motive force $N_2 \times I_2$. However, in practice there exist leakage flux ϕ_{k1} of the primary winding and ϕ_{k2} of the secondary winding. As a result of that, the leakage inductance $\mathbf{L_k}$ can be measured by short circuit at the secondary side.



$$L_{k1} = \frac{N_1 \cdot \Delta\phi_{k1}}{\Delta I_1} \qquad L_{k2} = \frac{N_2 \cdot \Delta\phi_{k2}}{\Delta I_2}$$

$$L_k = L_{k1} + \frac{1}{r^2} L_{k2}$$



- **The efficiency η of power transformers**

$$\eta = \frac{P_o}{P_{in}} = \frac{P_o}{P_o + P_t}$$

$$P_t = P_c + P_w = \mathbf{I_c} \cdot \mathbf{E_s} + \left(I_{in}^2 \times R_{w1} + I_2^2 \times R_{w2}\right)$$

$$P_o = r\mathbf{E_s} \cdot \mathbf{I_2}$$

$$\mathbf{I_{in}} = \left(\mathbf{I_c} + r\mathbf{I_2}\right) + \mathbf{I_m}$$

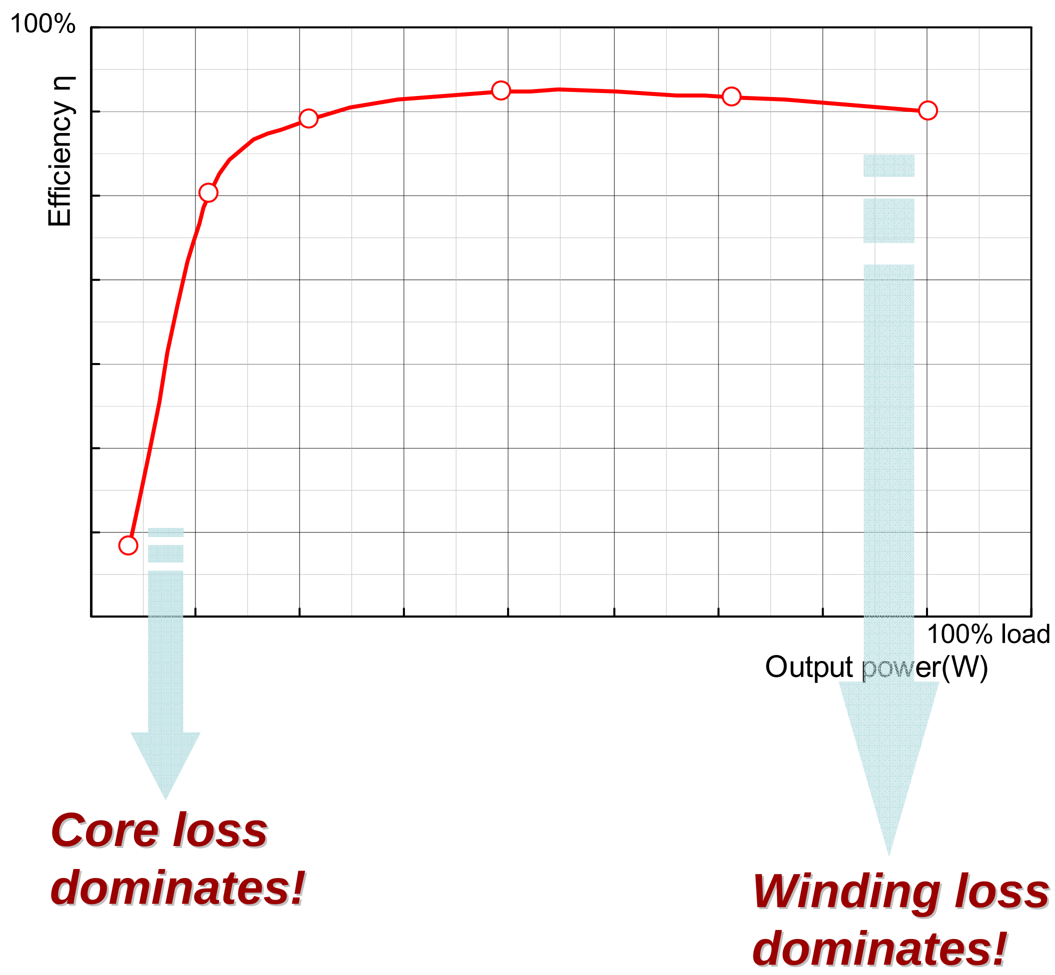
P_o: Output power

P_{in}: Input power

P_t: Total transformer loss

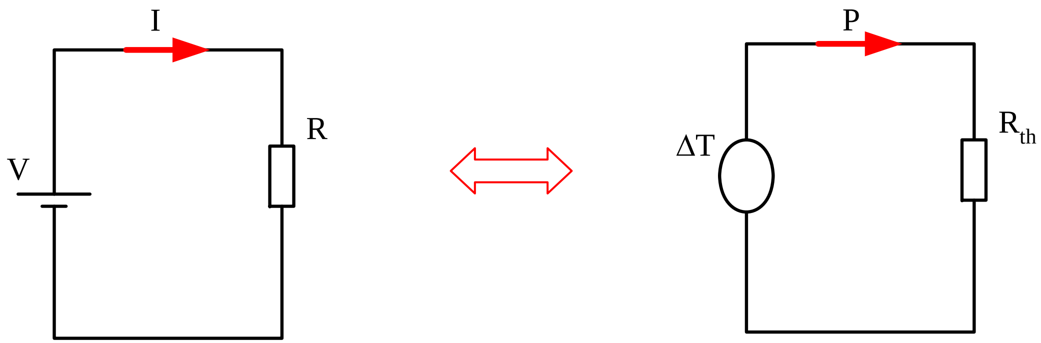
P_c: Core loss

P_w: Winding loss



• Temperature rise estimation of power transformers

Thermal resistance model in analogy to electric circuit:



Ohm's law: $V = I \times R$

Fourier's law: $\Delta T = P \times R_{th}$

, in which ΔT is the temperature rise, P is the heat transfer rate or power loss and R_{th} is the thermal resistance of the device.

It should be noted that the heat will in general be transferred to the surroundings by means of *conduction*, *convection* and by *radiation*, and each of these processes may be natural or may be artificially added. As a result of that, the concerned thermal resistance R_{th} will be dependent. One can refer to the table 9.3 of Snelling's book* to get R_{th} for temperature rise estimation.

** E.C. Snelling, Soft Ferrites – Properties and Applications, 2nd Edition, Butterworths, London (1988)*

Thank you for listening!